Combinatorics Exercises: Solutions

**Problem** 1:

5 tasks – critical

How many orders can you complete the tasks before the day ends?

**Solution**: Use permutations since we are arranging the entire set of elements. The total number of permutations is 120.

**Problem** 2:

Imagine your company is trying to gain customers by running an online ad campaign. The idea is to focus on a certain demographic which frequently uses social media. Your campaign will run ads on Facebook, Messenger, Instagram, Twitter and Reddit. Your graphical designers have created 8 different versions of the banner you can use. Based on this information:

5 different platforms.

8 different banners.

**a**) Calculate how many different options you have for the entire campaign, assuming you want to use a different one for each platform.

No repeated banners across all 5 platforms. Order doesn’t matter. Use non-repetition combinations. **Solution**: 56 different options

This is a variations problem. Variations are also the number of ways of putting k distinct balls into n distinct boxes such that each box receives at most one element. Therefore, the **solution** to a non-repetition variation is: 6720 different options.

**b**) Calculate how many different options you have for the entire campaign, assuming you can use the same banner for some or all the platforms.

Repeated banners allowed. Order doesn’t matter. Use repetition combinations. **Solution**: 792

Once again, we are mapping 1-1 distinct elements into distinct categories making this a variation problem with repetition. **Solution**: 32,768 different options

**c**) Calculate how many ways we can pick which of the 8 banners to use, assuming we use different ones for each platform.

This is combinations because order doesn’t matter. No repetition. **Solution**: 56 different ways.

d) Calculate how many ways we can pick which of the 8 banners to use, assuming we can use each one multiple times.

This is combinations with repetitions. **Solution**: 792 different ways.

**Problem** 3

You are renovating your entire apartment and want to repaint the walls of each room. The flat consists of two bedrooms, a kitchen, a living room, a bathroom, a study and a hall, or 7 rooms in total. You have at your disposal several colors of paint: white, yellow, orange, red, purple, blue, green, grey and pink.

7 rooms.

9 colors.

How many different ways can you paint the house, assuming…

a) …you paint all the rooms in different colors?

No repetition. 1-1 mapping. Variations. Solution: 181,440 different variations.

b) …you paint the bathroom, study and hall in white?

This reduces the amount of rooms that can have a different color by 3, so 4 total rooms. The problem doesn’t specify that the remaining 4 rooms cannot be painted white. Therefore, we still have 4 rooms and 9 colors with possible repetitions and a 1 to 1 mapping. **Solution**: 6,561 variations.

Note: The instructor clarified the intent of the question was non-repetition variation with 4 rooms and 8 colors which makes the official solution 1,680 variations. However, both answers are correct.

c) …you paint the two bedrooms in identical color?

Here the amount of rooms has decreased by 2 and based on the previous problem colors have decreased by 1 as well. Non-repetition variation solution: 6,720.

If we paint the two bedrooms in the same color, we can think of them as a single big room.

Thus, the number of rooms becomes 6 instead of 7. Unlike the previous problem, this problem apparently wants to assume that repetitions are allowed this time. I personally don’t understand why in the above problem when we painted 3 rooms an identical color (white) we reduced the total rooms by 3, but here we only reduce the total rooms by 1. This question and its answer are ambiguous. But the **intended solution** is repetition variations. 531,441 variations.

d) …you can only use grey and yellow?

7 rooms, 2 colors, repetition allowed. 128 variations.

**Problem** 4

This year, you are helping organize your college’s career fest. There are 11 companies which are participating, and you have just enough room fit all of them. How many ways can you arrange the various firms, assuming…:

N = 11

P = 11

a) … no firm has any preference where they want to be positioned?

Arrange the entire set of elements. Use permutations. **Solution**: 11! = 39,916,800 different arrangements.

b) … JP Morgan representatives made a deal, where they must be located exactly in the middle?

Here N = 10 and P = 10 because JPM is fixed. No repetitions and a 1-1 mapping of unique entities to distinct spots. So non-repetition variation. **Solution**: 3,628,800 arrangements

c) … JP Morgan, Citi Bank and Morgan-Stanley must be positioned in the middle 3 spots?

This is similar to the question above where N = 8, P = 8 with no repetitions and a 1-1 mapping. However, this also includes the different permutations that the 3 banks can form in the middle 3 spots for each variation. So the **solution** is the product of V(8,8) and P(3) = 241,920 arrangements

d) … Deutsche Bank representatives cancel, so you can give the additional space to one of the other companies?

N = 11 spaces

P = 10 firms

One firm gets 2 spaces. This is another compound solution. No repetitions variation and permutation. V(11,10) \* P(10)

There are 10! Arrangements of the firms. For each arrangement there are also 10 options for who gets the additional space. Therefore, the **solution** is 10! \* 10 = 36,288,000

**Problem** 5:

Your best friend is organizing a birthday party for her twins – Amy and Steve - and she put you in charge of ordering the cakes. The bakery offers several types of cakes – a Cheesecake, Sacher Cake, a Chiffon Cake, a Coconut Cake and a Carrot Cake. How many different ways can you order the cakes, assuming that…

N = 5 cakes

P = 2 people

a) … both twins enjoy all the 5 types of cake?

Variation with repetition since ordering two of the same cake is an option. **Solution**: 25 variations.

Since it doesn’t matter which of the two twins gets which cake in each order, we should use combinations. Therefore, the **solution** is combinations with repetitions: 15 combinations

b) … Steve dislikes Coconuts?

Here we have two separate cases. Steve can get any one of 4 cakes and Amy can get any one of 5 cakes. So, the **solution** is 20 combinations

\*Apparently the solution wants you to reduce the cake amount by 1 and perform a combination to get 10 combinations. But I disagree. Just because Steve doesn’t like coconut doesn’t mean Amy can’t have and enjoy a coconut cake. It doesn’t say each twin must eat the cake of the other anywhere, so I don’t see why Amy should have a limited selection. Therefore, I stand by my answer.

c) … Amy loves chocolate (Sacher)?

This limits one of the cakes to Sacher only. If repetitions, then there are 5 combinations since Steve can have any type of the remaining 5 cakes. If there are no repetitions, then Steve is constrained to 4 remaining cakes and there are only 4 possible combinations.

d) … each cake comes with a generic “Happy Birthday!” wish?

This is just an added arbitrary description to each cake and doesn’t affect the outcome at all. Therefore, the **solution** is the same a) 15 combinations

e) … each cake comes with a personalized “Happy Birthday Steve!” or “Happy Birthday Amy!” sign?

In this case, order matters since each cake is now unique in its description. We use variations with repetitions. The **solution** is: 25 variations.

**Problem** 6

You want to go to the gym between lectures every day, but you only have an hour to workout.

Knowing this, you decide to do a circuit workout.

Your start with 5 minutes of cardio as a warm-up, then you hit two different leg exercises, followed by a chest exercise, as well as one for shoulders. After that, you continue with a bicep exercise and a triceps one, before moving to the back one. You finish the circuit with 2 abdominal exercises like a plank and some crunches.

After completing the circuit several times, you end with another 10 minutes of cardio before you stretch and leave.

Now, assuming the gym has ellipticals, treadmills and stationary bikes, you have 3 options for the cardio.

Additionally, you have 5 different leg exercises you can do.

You have 4 choices of what to do for each of the next 3 muscle groups (chest, shoulders and bicep).

For triceps you have heavy preferences towards two specific exercises, so you always do one of the two.

The same can be said about the back.

When it comes to the abdominal exercises, you have 4 options once again.

Taking everything into consideration, if you want to do a different workout each day, how long will it take you to run out of options?

Organize what we have:

Cardio: 3 Exercises x 2 Sets – Repetitions allowed

Legs: 5 Exercises x 2 Sets – No Repetitions

Chest: 4 Exercises x 1 Set

Shoulders: 4 Exercises x 1 Set

Bicep: 4 Exercises x 1 Set

Triceps: 2 Exercises x 1 Set

Back: 2 Exercises x 1 Set

Abs: 4 Exercises x 2 Sets – No Repetitions

This is a combination problem with multiple separate sample spaces. The final solution will be the product of the combinations of each sample space.

Cardio: 9 variations

Legs: 20 Variations

Chest: 4 combinations

Shoulders: 4 combinations

Biceps: 4 combinations

Triceps: 2 combinations

Back: 2 combinations

Abs: 12 Variations

**Solution**: 552,960 different variations.